

Constraints on dark energy from holography

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Using the holographic principle we constrained the Friedmann equation, modified by brane-cosmology inspired terms which accomodate dark energy contributions in the context of extra dimensions.

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More than twenty years ago, Bekenstein first proposed that for any isolated physical system of energy E and size R , there is an upper bound on the entropy $S \leq S_B = 2\pi RE$ [1]. In spite of criticisms about that derivation [2], the Bekenstein entropy bound has received independent support [3]. Choosing R to be the particle horizon, Bekenstein gave a prescription for a cosmological extension of the bound [4].

In view of the well-known example of black hole entropy, an influential holographic principle was put forward ten years ago, relating the maximum number of degrees of freedom in a volume to its boundary surface area [5]. For systems of limited gravity, the Bekenstein entropy bound implies the holographic principle. The extension of the holographic principle to cosmological settings was first addressed by Fischler and Susskind (FS) [6]. Subsequently, various different modifications of the FS version of the holographic principle have been raised [7, 8]. In addition to the study of holography in homogeneous cosmologies, attempts to generalize the holographic principle to a generic realistic inhomogeneous cosmological setting were carried out [9, 10]. Other extensions of the holographic principle to brane cosmology have also been given [11].

The holographic principle is viewed as a real conceptual change in our thinking about gravity [12]. It is interesting to note that holography implies the probable values of the cosmological constant in a large class of universes [13]. In the inhomogeneous cosmology, holography was also realized as a useful tool to select physically acceptable models [10]. Recently, the idea of holography has further been applied to the study of inflation and gives plausible upper limits to the number of e-foldings

[14].

Recent observations based on type IA supernova as well as from Cosmic Microwave Background Radiation results imply that the expansion of our universe is speeding up instead of slowing down [15]. This discovery has posed a fundamental challenge to the standard models of both particle physics and cosmology. Whether holography can bring us some light in understanding the profound puzzle posed by dark energy is a question we want to ask. Motivated by the suggestions in [13] [16], attempts to apply the holography to the dark energy due to quantum corrections to the cosmological constant were carried out in [17] [18]. It was found intriguing that using the event horizon size as the infrared cutoff relevant to the dark energy, the equation of state of the dark energy can be predicted correctly [18].

In addition to attributing dark energy to quantum vacuum energy, to a light and scalar field or a frustrated network of topological defects, recently further attention has been paid to a modification of general relativity in order to explain the accelerated expansion [19, 20, 21]. This idea is attractive since it has close ties to high energy physics such as string theory or extra dimensions and does not necessarily suffer from fine tuning problems. In this paper, we will parametrize the physical cause of acceleration with an arbitrary additional term in the Friedmann equation introduced in [20]. Employing the idea of holography, we are going to derive the constraints on the equation of state of dark energy and the extra-dimensional contribution to the Friedmann equation.

Suppose that the effects of extra dimensions manifest themselves as a modification to the Friedmann expansion rate equation of the form [20]

$$(H/H_0)^2 = (1 - \Omega_M)(H/H_0)^\alpha + \Omega_M(1+z)^3. \quad (1)$$

The first term on the right-hand-side of (1) is the correction to Friedmann equation due to infinite extra dimensions accounting for the geometric dark energy. H_0 is the

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Hubble parameter at present, Ω_M is the critical energy density of matter and z is the redshift relating to cosmic scale factor a by $z = a^{-1} - 1$.

The model under consideration describes an ever expanding universe. Although a further transition is not ruled out, this is a good simplifying assumption. A cosmological constant with dust and radiation also leads to eternal expansion.

As an example, consider a simple, single extra-dimensional model [19]. The effective, low energy action is given by

$$S = \frac{M_{Pl}^2}{r_c} \int d^4x dy \sqrt{g^{(5)}} \mathfrak{R} + \int d^4x \sqrt{g} (M_{Pl}^2 R + L_{SM}), \quad (2)$$

where $M_{Pl}^2 = 1/8\pi G$ is the four-dimensional Planck scale, $g_{AB}^{(5)}$ is the five-dimensional metric, y is the extra-dimension, \mathfrak{R} is the five-dimensional Ricci scalar, g is the trace of the four-dimensional metric, R is the four-dimensional Ricci scalar and L_{SM} is the Lagrangian of the fields in the standard model. For the Friedmann-Robertson-Walker Ansatz

$$ds_5^2 = f(y, H) ds_4^2 - dy^2, \quad (3)$$

where ds_4^2 is the four-dimensional maximally symmetric metric, and H is the four-dimensional Hubble parameter, one gets the modified Friedmann equation on the brane under the form

$$H^2 \pm \frac{H}{r_c} = \frac{8\pi G \rho_m}{3}. \quad (4)$$

These general features persist for an arbitrary number of dimensions, leading to (1).

The general form (1) is mathematically equivalent to a time variable dark energy equation of state function [20] [21]

$$\omega_D(z) = -1 + \frac{1}{3} \frac{d \ln(H^\alpha/H_0^\alpha)}{d \ln(1+z)} + \frac{1}{3} \frac{\dot{\Omega}_M}{1 - \Omega_M}. \quad (5)$$

As a matter of fact $\Omega_M = \rho/\rho_c$ is not a constant, but it can be neglected. Indeed, from $\dot{\rho} = -3H(1+\omega)\rho$, we have

$$\rho = \rho_0 \exp[3(1+\omega) \int_a^1 d \ln a], \quad (6)$$

where we have taken ρ_0 as the matter density at present, $a(t_0) = 1$. For $\omega = 0$, the matter dominated era, we have $\rho = \rho_0 a^{-3}$. Then $\rho_0/\rho_c = \rho a^3/\rho_c$. Considering $\rho_c = 3M_p^2 H^2$, we have $\frac{3M_p^2 H_0^2}{3M_p^2 H^2} = \Omega_M a^3$, then $\Omega_M = H_0^2 H^{-2} a^{-3} = H_0^2/(a\dot{a}^2)$, which depends on time.

However such an additional term is very small from now until the distant future. Taking $a \sim t^p$, ($p = 2/(1+\omega)$), for the expansion of the universe due to matter), the additional term is proportional to $1/t^{3p-1}$, which should disappear for large t . Therefore we can neglect the third

term of (5) here. For a distant future the last term in (5) will be even smaller and can be neglected for sure (unless we allow new physics in the future).

At this point it is worthwhile making some comments concerning the fact that in the distant future there is a further solution of the Friedmann equation given by de Sitter space, that is, an exponentially growing universe, for which our arguments fail. In fact, the arguments given in this paper are valid for a varying equation of state, which can change from some value in the past, to a value not equal to -1 today. In case it stabilizes in the future at the value -1, then the scale factor of the universe may evolve in different stages:

- matter dominated era, $a \sim t^{2/3}$, $\ddot{a} < 0$.
- after dark energy starts to play effect, the evolution of the scale factor will become faster, $a \sim t^{\alpha/[3(1+\omega_D)]}$, $\ddot{a} > 0$.
- when the dark energy dominates, neglecting the matter, the scale factor evolves exponentially.

We argue that we are living the second stage, where the dark energy plays the role of the expansion of the universe, the universe expands not as fast as that of the purely dark energy era, so we can use the expression of the scale factor indicated in the second item above.

If ω in the far future is not equal to minus one, the argument above stops in the second phase.

In any case, our paper relies on the hidden assumption that the ω parameter varies with time. This is a matter which is being discussed since some time [23].

The continuity equation still holds, $\dot{\rho} = -3H(\rho + P)$. The equation of state of the universe follows immediately,

$$\omega_T(z) = -1 + \frac{1}{3} \frac{d \ln(H^2/H_0^2)}{d \ln(1+z)}. \quad (7)$$

For an equation of state with a constant ω -parameter, $\omega = P/\rho$, the energy density varies as $(1+z)^{3(1+\omega)}$. It was found that during the matter dominated era, $\omega = -1 + \alpha/2$, while during the earlier radiation-dominated epoch, $\omega = -1 + 2\alpha/3$ [20].

Without dark energy, the universe expands as $a \sim t^{\frac{2}{3(1+\omega)}}$. For the far distant future, we can neglect all matter density ρ , thus we have from the Friedmann equation the exponential expansion of the universe. This corresponds to $\omega_D = -1$. For the present universe, the universe has not been expanded that fast, and we can assume ω_D seen today as a constant. Then due to the dark energy, the expansion of the universe $a \sim t^{\alpha/[3(1+\omega_D)]}$. To experience accelerated expansion, $\ddot{a} > 0$, which requires $\frac{\alpha}{3(1+\omega_D)} > 1$.

According to Bekenstein, for a system with limited self-gravity, the total entropy S is less or equal than a multiple of the product of the energy and the linear size of the system. In the present context, we can also examine the entropy bound of the dark energy and give the constraint of the model parameter α to explain the dark

energy at present. So the bound we obtain can be explained as the bound on the structure of the dark energy seen today. For the cosmological setting, the Bekenstein bound applies to a region as large as the particle horizon [4], which is defined by the distance covered by the light cone emitted at the singularity $t = 0$, $L_H = a(t)r_H(t)$, where $r_H(t)$ is the comoving size of the horizon defined by the condition $dt/a = dr_H$.

$$r_H = \int_0^t \frac{dt'}{a(t')} = \frac{3(1+\omega_D)}{3(1+\omega_D)-\alpha} t^{\frac{3(1+\omega_D)-\alpha}{3(1+\omega_D)}}. \quad (8)$$

The total entropy inside the particle horizon behaves as $S = \sigma L_H^3/a^3$, where σ is the entropy density measured in the comoving space, which is constant in time. The Bekenstein entropy is $S_B = EL_H = \rho L_H^4$. Thus, the ratio S/S_B reads

$$\frac{S}{S_B} \sim \left(\frac{3(1+\omega_D)}{3(1+\omega_D)-\alpha} \right)^{-1} \sigma t^{-\frac{1+(1-\alpha)\omega_D}{(1+\omega_D)}}. \quad (9)$$

In order to satisfy the Bekenstein bound, we require that $-1 < \omega_D < 1/(\alpha-1)$ for $\alpha > 1$; $\omega_D > -1$ or $\omega_D \leq -1/(1-\alpha)$ for $0 < \alpha < 1$ and $\omega_D \geq -1/(1-\alpha)$ or $\omega_D < -1$ for $\alpha < 0$. It is easy to see that for $\alpha > 1$, the constraint on the equation of state is too loose. It is over the range accounting for the dark energy.

Furthermore, the physical comoving size of the particle horizon should be positive, which requires $\frac{3(1+\omega_D)}{3(1+\omega_D)-\alpha} > 0$. This gives more constraints on the equation of state. For $\alpha > 1$, it leads to $-2/3 < \omega_D < 1/(\alpha-1)$, which fails to describe the dark energy. For $0 < \alpha < 1$, the range of dark energy state has been further refined to $\omega_D > -1 + \alpha/3$ or $\omega_D \leq -1/(1-\alpha)$. For $\alpha < 0$, we have $\omega_D \geq -1/(1-\alpha)$ or $\omega_D < -1 + \alpha/3$. However none of the above range of the equation of state can accommodate the accelerated expansion, $\ddot{a} > 0$.

We delve further in the problem by replacing the particle horizon by the future event horizon, $L_h = ar_h$, where $r_h(t) = \int_t^\infty \frac{dt'}{a(t')} = \frac{3(1+\omega_D)}{\alpha-3(1+\omega_D)} t^{\frac{3(1+\omega_D)-\alpha}{3(1+\omega_D)}}$. The ratio S/S_B now becomes

$$\frac{S}{S_B} \sim \frac{\sigma L_h^3/a^3}{\rho L_h^4} \sim \left(\frac{3(1+\omega_D)}{-3(1+\omega_D)+\alpha} \right)^{-1} \sigma t^{-\frac{1+\omega_D(1-\alpha)}{1+\omega_D}}. \quad (10)$$

In order to satisfy the Bekenstein bound, we require $-1 < \omega_D < 1/(\alpha-1)$ for $\alpha > 1$; $\omega_D > -1$ or $\omega_D \leq -1/(1-\alpha)$ for $0 < \alpha < 1$ and $\omega_D \geq -1/(1-\alpha)$ or $\omega_D < -1$ for $\alpha < 0$.

To keep the physical comoving size of the event horizon to be positive, the range of the equation of state is confined to $-1 < \omega_D < -1 + \alpha/3$ for $\alpha > 0$. To meet the observational result, we should take $\alpha \in (0, 1)$, so that $-1 < \omega_D < -2/3$. For $\alpha < 0$, $-1 + \alpha/3 \leq \omega_D < -1$.

Checking the condition for the speeding up expansion of our universe, it is found that both of the above ranges can accommodate accelerated expansion.

In cosmology, particle horizon (or event horizon) refers to the entire past (or future) light-cone. Unlike particle

or event horizons, the cosmological apparent horizon does not refer at all to either initial or final moment of the universe, furthermore it is observable. If we choose the apparent horizon as the boundary, the Bekenstein bound becomes

$$\frac{S}{S_B} \sim \frac{\sigma \tilde{r}_{AH}^3/a^3}{\rho \tilde{r}_{AH}^4} \sim \frac{\alpha}{3(1+\omega_D)} \sigma t^{-[1+\omega_D(1-\alpha)]/(1+\omega_D)}, \quad (11)$$

where $\tilde{r}_{AH} = 1/H$ has been taken. The requirement of satisfying the Bekenstein entropy bound together with the accelerated expansion lead to the same constraint on the parameter α and equation of state as taking event horizon as the boundary.

In the cosmological setting, it is usually believed that the Bekenstein bound is looser than the holographic bound, which is the opposite of what we understood for isolated system. It would be of great interest to investigate whether the holographic entropy bound can give tighter constraints on the equation of state and extra-dimensional contributions to the dark energy.

Directly applying the FS version of the cosmic holographic principle by using the particle horizon [6], we again face the problem that in the range for the equation of state that we obtained, the universe cannot experience accelerated expansion.

Extending the cosmic holography by considering that the entropy cannot exceed one unit per Planckian area of its boundary's surface characterized by the event horizon, we have

$$\frac{S}{A} \sim \frac{\sigma L_h^3/a^3}{L_h^2} \sim \sigma \frac{r_h}{a^2} \sim \frac{3(1+\omega_D)}{\alpha-3(1+\omega_D)} \sigma t^{1-\frac{\alpha}{(1+\omega_D)}}. \quad (12)$$

Combining the requirement that the ratio S/A not increasing with time, positive r_h and accelerated expansion $\ddot{a} > 0$, we obtain $-1 < \omega_D < -1 + \alpha/3$ for $\alpha > 0$. To meet observation, α is further refined to $0 < \alpha < 1$, then $-1 < \omega_D < -2/3$. For $\alpha < 0$, we have $-1 + \alpha/3 < \omega_D < -1$.

If we choose the apparent horizon as the boundary, the holographic principle should be expressed as: the entropy inside the apparent horizon can never exceed the apparent horizon area [24]

$$\frac{S(t)}{A} = \frac{\sigma Vol_{AH}(t)}{A(t)} \leq 1 \quad (13)$$

where $Vol_{AH}(t) = \frac{V(r_{AH}(t))}{a^3(t)}$ denotes the comoving volume inside the apparent horizon. Eq(13) can be rewritten as

$$\frac{S}{A} \sim \frac{3(1+\omega_D)}{\alpha} \sigma t^{1-\alpha/(1+\omega_D)} \quad (14)$$

The requirement that the ratio S/A does not increase with time together with the accelerated expansion leads us to the same constraints for the equation of state of dark energy as we got by using the event horizon. Choosing the apparent horizon leading to a reasonable equation

TABLE I:

Boundary	Bekenstein Bound	Holographic Bound
Particle hor.	Not satisfied	
Event hor.	<i>Satisfied</i> , $-1 < \omega_D < -2/3$ (for $0 < \alpha < 1$); $-1 + \alpha/3 < \omega_D < -1$ (for $\alpha < 0$)	
Apparent hor.	<i>Satisfied</i> , $-1 < \omega_D < -2/3$ (for $0 < \alpha < 1$); $-1 + \alpha/3 < \omega_D < -1$ (for $\alpha < 0$)	

of state of dark energy is different from the discussion in [17], a consequence of the special dark energy model we have supposed.

We would also like to extend the discussion by using the Hubble entropy bound. Following the usual holographic arguments, one then finds that the total entropy should be less or equal than the Bekenstein-Hawking entropy of a Hubble size black hole times the number n_H of Hubble regions in the universe. The entropy of a Hubble size black hole is roughly $HV_H/4$, where V_H is the volume of a single Hubble region. Considering the universe with volume V , one obtains an upper bound on the total entropy $S < S_H = HV/4$ [8]. If one applies the Hubble entropy bound to a region of size L_H , it is interesting to find $S_H = (S_{BSFS})^{1/2}$.

Employing the Hubble entropy bound to the region of size a , the ratio

$$\frac{S}{S_H} \sim \frac{4\sigma a^3}{HV} \sim a/\dot{a} \sim t \quad . \quad (15)$$

Thus with the assumed domination of the gravitational correction in the Friedmann equation, Hubble entropy bound will be violated. This is not surprising, since not like the Bekenstein bound and the FS bound, the Hubble entropy bound is appropriate in the strong self-gravitating universe $Ha > 1$ [8]. With the extra-dimensional contribution to the dark energy to account for the accelerated expansion, the universe will be with limited self gravity.

In summary, we have used the idea of holography to study the constraints on the geometric dark energy. Our results are listed in the table. Contrary to the original understanding in the cosmological setting, we found Bekenstein entropy bound and holographic entropy bound play the same role in refining the geometric dark energy. To account for the dark energy, α cannot be bigger than unit. For $0 < \alpha < 1$, the equation of state of the extra-dimensional contribution lies in the range $-1 < \omega_D < -2/3$, which can be treated as dark energy. For $\alpha < 0$, the extra-dimensional effect acts as an effective phantom energy, where $-1 + \alpha/3 < \omega_D < -1$. The failure by using the particle horizon to explain the accelerated expansion of our universe due to geometric contribution to the dark energy has got independent support in [18], where the dark energy was attributed to the cosmological constant. The reason for such a failure is that the particle horizon is related to the early universe, when dark energy played no role.

After our work was finished, we noticed that observational constraints on a modified Friedmann equation which mimics the dark energy was studied in [22]. Our holographic constraints on α got independent support from their Supernovae Type IA and CMB study. $\alpha = 0$, which is excluded in our holographic investigation, is also disfavoured in the studied parameter space in [22]. Combining CMB and SNIa, in [22] they got tighter constraint on α , especially the lower bound on the negative value of α , which calls for further understanding in our study. Comparing with the investigation of the observational constraint, we found that holography again plays a powerful role in the study of gravity.

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